Exercise sheet 5: Recursive functions

1. Show that functions in **PRF** are total (*cf.* Theorem 18).

2. Let $\{f_k \mid k \in \mathbb{N}_0\}$ be the set of Ackermann functions as saw in theory. Show that

$$\forall k \in \mathbb{N}_0,\ x > x' \Rightarrow f_k(x) > f_k(x')$$

3. Define the following function as **RF**. $f(x) = \lfloor \sqrt{x} \rfloor$

4. Let $div(x, y) = \lfloor \frac{x}{y} \rfloor$.

   (a) Define the function div as **RF**, assuming $0/0 = 0$

   (b) Define the function div as **RF**, assuming $0/0$ is undefined.

   (c) Define $mod(x, y)$ which outputs the rest of the division of x by y, as **RF**using the definition of div.

5. Let $minus(x, y) = \begin{cases} x & \text{if } y = 0 \\ Pd(minus(x, Pd(y))) & \text{if } y \geq 1 \end{cases}$.

   (a) Calculate $f(4)$ where f is defined as $\mu_y(minus(x, Pd(y)))$.

   (b) Is f a total function or a parcial one?

6. Prove that the function f_0 from the Ackermann sequence majorates all the basis function. (*cf.* Theorem 20).